The Distributional Implications of Group Lending

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Abstract

In recent years, the value of joint liability and social networks in expanding credit outreach has been questioned and several prominent microfinance institutions have initiated a shift from group loans to individual contracts with members of groups. The purpose of our paper is to build on the theoretical work on group lending to better understand links between contractual structure, credit outreach and borrower welfare. We focus on the *ex-post* moral hazard problem of repaying a loan on completion of a successful self-employment project and show that in the absence of social sanctions, the largest available loans are offered under individual and not group contracts. Social sanctions within groups can, under certain conditions, improve enforcement and outreach but they cannot, in general, substitute for bank sanctions. For those that benefit from group lending, we show that welfare gains are increasing in initial wealth.

1 Introduction

The ideology and practice of poverty alleviation has been deeply influenced by the idea that group lending can empower the poor by providing them access to credit. The Grameen Bank of Bangladesh first popularized group loans in the 1970s. Loans were made to groups of 5 members and borrowers were made jointly responsible for repayment. Default by any member made

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the others ineligible for future loans. It was believed that joint liability combined with social sanctions would induce borrowers to repay loans and thereby constitute a financially sustainable model of lending and self-employment among *the poorest of the poor*. Similar approaches were subsequently developed by literally hundreds of organizations across the world.

With the successful expansion of group lending in the nineties there emerged a theoretical literature which linked the features of microcredit contracts to repayment incentives. Borrower selection, peer monitoring, risk-pooling and social sanctions were shown to be potential mechanisms which allowed groups to mitigate some of the serious and well known informational and enforcement problems that afflicted credit markets (Stiglitz, 1990; Banerjee et al., 1994; Ghatak, 1999). This literature has been surveyed in Ghatak and Guinnane (1999) and Armendariz de Aghion and Morduch (2005).

More recently, the value of joint liability and social networks in expanding credit outreach has been questioned by both researchers and practitioners. The very poor seem to be underrepresented in credit groups (Morduch, 1988) and appear to leave them at higher rates (Baland et al., 2008). The impact on repayment rates is also ambiguous because the financial distress of a few members increases the burden on the others who may in turn decide that group default is preferable. This ambiguity has been theoretically examined by Besley and Coate (1995) and it is illustrated emprically by Giné and Karlan (2009) who randomize assignment of individual and joint liability regimes in different regions in the Philippines and find no significant difference in default rates.

On the field, several prominent microfinance institutions have initiated a shift from group loans to individual contracts with members of groups. In 2002, the Grameen Bank replaced their hallmark model of group lending with Grameen II, which eliminated joint liability while maintaining the group structure to foster solidarity. The introduction of the new system appears to have brought back many former Grameen members and the total number of borrowers has increased from 3 to 8 million (Figure 1).[1] Particularly interesting is the admission by the bank that very poor individuals are often best served outside groups (Grameen Bank, 2009):

> A destitute person does not have to belong to a group...Bringing a destitute woman to a level where she can become a regular member of a group will be considered as

[1]Wright et al. (2006) consider membership until 2005 and report that "Grameen took 27 years to reach 2.5 million members- and then doubled that in the full establishment of Grameen II". The Grameen Bank's own website currently reports membership of about 8 million.

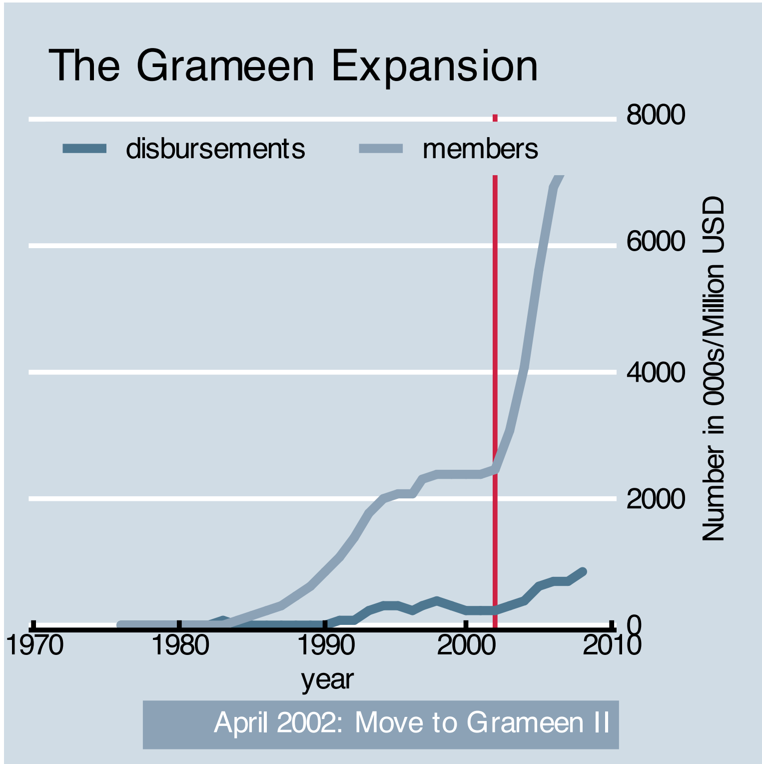


Figure 1: Grameen Bank Lending Under Alternative Contractual Regimes

a great achievement of a group.

This trend is by no means universal and the microfinance sector continues to be institutionally diverse. In India, for example, *Self Help Groups* which adhere fairly strictly to the joint-liability contracts constitute 73 per cent of borrowers in the micro-finance sector[2] and micro-finance institutions often combine individual loans with the traditional group lending model.[3]

The purpose of our paper is to build on the theoretical work on group lending to better understand links between contractual structure, credit outreach and borrower welfare. As shown in the literature, there are many plausible mechanisms through which joint-liability can affect the performance of microcredit groups. We focus on the *ex-post* moral hazard problem of repaying a loan on completion of a successful project. We model a simple investment game in which individuals can invest in a project of fixed size and are offered either individual contracts or

[2]See Srinivasan (2009), p 5, Table 1.2

[3]Ghate (2008) Table A.2 contains a classification of 129 recognized Micro-finance institutions all over India. Many of these are reported to offer both individual and group contracts

group loans by a competitive banking system. Banks can impose a fixed non-pecuniary sanction on defaulting borrowers. With group loans, all members are sanctioned by the bank if the total amount borrowed by the group is not repaid. In addition, there may be social sanctions within a group.

We begin with a characterization of credit market equilibrium under the two types of contracts. Our first result shows that in the absence of social sanctions, the largest available loans are offered under individual and not group contracts. This is because repayment incentives under both group and individual liability are constrained by the same level of bank sanctions and under joint liability, successful borrowers have to compensate the banking system for failures within their group as well as for all groups which do not succeed in repaying their loans. This result is significant because it shows that joint liability *per se* cannot bring credit to the doorstep of those not currently served by the banking system. If outreach is greater under group lending, it is other features of these groups - social networks, sanctions and better project choices- that allow this sector to reach the poor.

Social sanctions within groups can improve enforcement and outreach but we show that their effects depend critically on the nature of project uncertainty. When the level of uncertainty is very high, social sanctions effectively pool risks and can always improve outreach. In contrast, and more surprisingly, social sanctions can turn out to be ineffective when projects become less risky. The reason for this is that, given an expected project return, less risky projects yield less when successful and even arbitrarily large social sanctions can at most extract the full income from the project.

We also compare the potential of bank and social sanctions to improve repayment incentives and credit outreach. This is related to a well established debate in the development literature on the extent to which informal networks can substitute for market incentives in the presence of weak legal systems. We consider the implications of changing the mix of bank and social sanctions, given a fixed value of total sanctions that can be imposed on an individual. We show that while social sanctions can, at best and under limited conditions, substitute for bank sanctions, an increase in bank sanctions always increases outreach for both individual and group loans. For those already receiving group loans an increase in both types of sanctions raises welfare and these gains are concentrated among poorer borrowers.

We then turn to the welfare consequences of group lending. For borrowers requiring small loans, both types of contracts are offered by banks. We compare the welfare of given borrowers under group and individual loan contracts. Group loans can improve welfare by lowering the probability of being sanctioned by the bank. We show that, for fixed group size, the greatest welfare gains are for groups with the highest initial endowment of wealth. This happens because groups that require smaller loans also need fewer successful projects to repay them and the possibility of pooling risk is therefore greatest for these groups.

Finally, the existing literature on group lending has focussed almost entirely on two person groups. We consider gains from group size as a function of borrower wealth and find that this relationship is not monotonic. For high wealth (and small loans) larger groups are able to pool risks and optimally sized groups always have more than two members. For low wealth levels a fall in group size can increase borrower welfare.

Taken together, these results show that joint liability can increase borrower welfare but its ability to provide the poor access to credit depends on a complex set of factors and is often limited. Ultimately, the poorest households may be best served by providing them individual loans on more favorable terms and promoting alternative programs of poverty alleviation.

2 The model

We begin with a standard model of the credit market with wealth heterogeneity. Our principal unit of analysis is a set of households in a community.[4] Communities are of size n and contain households of homogeneous wealth w.[5] Community wealth varies according to some continuous distribution and we examine the role of credit contracts in mediating the relationship between wealth, investment and earnings.

Each household is risk-neutral and can choose to engage in a self employment project which requires indivisible investment of one unit and yields a return ρ with probability π and zero otherwise. There are no other project costs.[6]

[4] We use the terms households and individuals interchangeably.

[5] Most group-lending programs have tried to ensure that members within the group are similar in in terms of their initial endowments. At the start of the Indian microcredit program, for example, non-government organizations were issued guidelines by the Reserve Bank of India explicitly suggesting that they foster savings and credit groups among households of similar social standing.

[6] Costs of effort are easily incorporated. In our model the project returns can be interpreted as being net of these costs.

A competitive banking sector takes deposits and lends to individuals or to groups under joint liability contracts. Households that do not invest can obtain a risk-free gross return of r, which is also the opportunity cost of bank funds. If loans are not repaid, banks can impose a non-pecuniary sanction K on the borrower. In the case of non-repayment by a group, each member faces K. These sanctions provide no direct benefits to the bank and have no associated costs. Banks cannot directly observe project success and contract consists of a loan size, an interest rate and the bank sanction K.

To focus on the role of joint liability in determining the nature of group loans, we abstract from questions of group formation and monitoring that have been studied by others. We take as given a fixed group size n and think of group projects as individual projects undertaken by each member of the group.

In addition to bank sanctions, group loans may be subject to social sanctions. We follow the existing literature on group lending in assuming that the maximum level of these sanctions is exogenously given. Social sanctions inflict a utility cost γ on sanctioned household and are costless to others in the group. These could, for example, represent a lost reputation. In practice, the costs and motives associated with sanctions within social networks are varied and complex. Our purpose here is only to explore the extent to which any cost of this kind can improve repayment incentives within groups.

The sequence of actions is as follows. A group with wealth w and n members may receive a loan providing each member $(1 - w)$ to invest in the project. Project returns are realized and observed by group members. Successful members then simultaneously announce contributions towards bank repayment. If available contributions are high enough, repayment is made. If not, the bank sanctions each member K and non-contributing members may also be subject to social sanctions γ.

2.1 Individual Loans

What is the largest loan an individual can get from the bank? Those with failed projects pay nothing, so if all successful borrowers repay, banks break-even by charging $\frac{r}{\pi}$. Borrowers prefer repayment to bank sanctions as long as $\frac{r}{\pi}L \leq K$. The largest loan available under an individual contract is therefore

$$L_i = \frac{\pi K}{r} \tag{1}$$

and the lowest wealth level at which individuals will be able to borrow enough for investing in the project is

$$w_i = \min(0, 1 - L_i)$$

If $w_i = 0$, then all households have access to credit. Since we are interested in the problem of outreach, we assume that parameter values are such that $w_i > 0$ or equivalently, $L_i < 1$. This implies $K < \frac{r}{\pi}$.

For an individual of wealth w, the expected utility of taking a loan of size $L = 1 - w$,, with $L \leq L_i$ is given by:

$$U_i = \pi \left(\rho - \frac{r}{\pi} L \right) - (1 - \pi)K - rw = \pi\rho - r - (1 - \pi)K \tag{2}$$

For any household to invest in the project with borrowed funds, expected returns must be high enough for U_i to be positive. This requires investors being compensated for the opportunity cost of funds as well as the sanctions they face in the case of project failure. We define $\bar{\rho}$ as the minimum return on successful projects:

$$\bar{\rho} = \frac{1 - \pi}{\pi} K + \frac{r}{\pi}. \tag{3}$$

Throughout the paper we assume a project return $\rho \geq \bar{\rho}$.

2.2 Group Loans

Under joint liability, incentives to repay depend on the number of successful members in the group and their repayment strategies. As in most such coordination games, there are many equilibria. Since we are interested in maximal outreach, we restrict our attention to the symmetric repayment equilibrium with the smallest positive contributions by each member. This assumes away coordination problems within the group.

In the case of individual contracts, all loans that get made by the bank have the same probability of repayment, π and the same interest rate, $\frac{r}{\pi}$. With joint liability, failures can be subsidized by successes within the group and, since smaller loans require fewer successes, interest rates for group loans are increasing with loan size.

Let $B(n, j, \pi)$ be the probability of j or more successes in n Bernoulli trials with a probability of success on each trial equal to π. We refer to this by $B(j)$ if there is no ambiguity about the n and π in question. Now consider the following expression for each value of $j = 1, 2, \ldots n$:

$$\frac{n}{j}\frac{r}{B(j)}L$$

Suppose that repayment on a per member loan of L occurs if there are a minimum of j successes in the group. $B(j)$ then represents the probability of group success. In this case banks would charge an interest rate of $\frac{r}{B(j)}$ and the above expression represents payments per successful member.

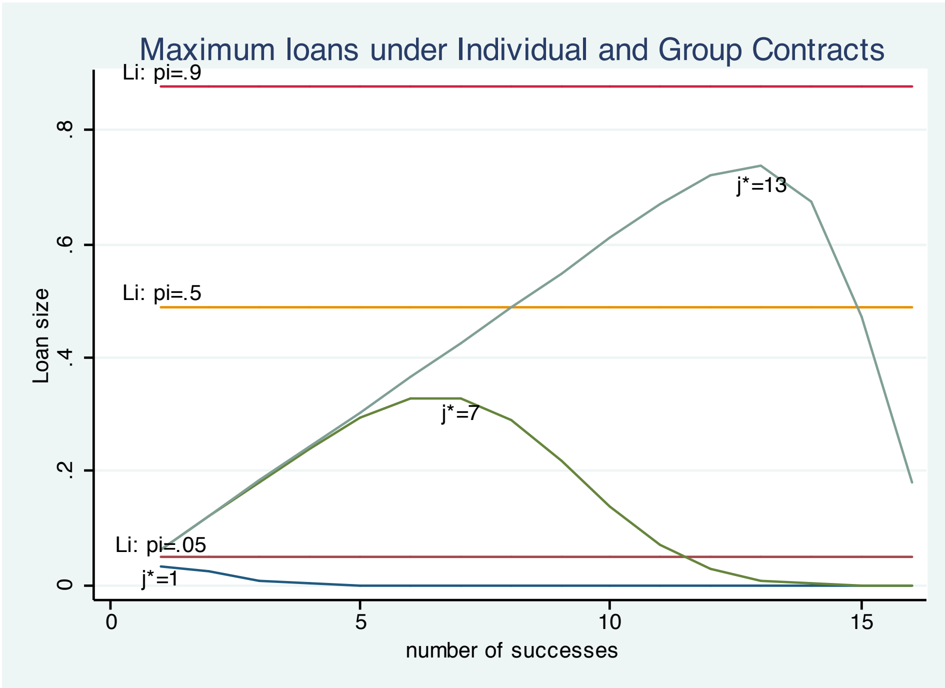
Repayment in turn depends on incentives, as captured by sanctions K and γ, as well as the amount available from a successful project, ρ. Total sanctions $K + \gamma$ determine repayment incentives, but since payments are at most equal to the total return of a successful member, ρ, social sanctions above $\rho - K$ have no effect on repayment. A per-member loan of size L is therefore feasible if there exists a value of j such that

$$\frac{n}{j}\frac{r}{B(j)}L \leq \min(\rho, K + \gamma). \tag{4}$$

If there are multiple values of j for which (4) is satisfied for a given loan size L, the smallest such value determines the group contract because it corresponds to the lowest interest rate. A group contract is therefore of the form $(L, \frac{r}{B(j(L))}, K)$ where $j(L)$ is the smallest number of successes required for repaying a per member loan of size L. The largest group loan L_g is the value of L for which (4) holds with equality.

While this means that groups requiring smaller loans will be charged lower interest rates because they require fewer successes for repayment, the largest group loans L_g cannot require a value of j which far exceeds the expected number of successes for individuals in the population. This is because, as j increases above its expected value, the probability of achieving j successes drops enough to cause $jB(j)$ and therefore the maximum loan size to fall.

The figure below shows maximum loan sizes under individual and group contracts for three different levels of project uncertainty, namely $\pi = \{.05, .5, .9\}$ for group size $n = 16$. In each case, feasible group loans are maximized at a value of j below $n\pi$. The following lemma establishes that this is indeed always the case.



Lemma 1. *The function $jB(n, j, \pi)$ starts at zero, takes the value $n\pi^n$ at $j = n$ and attains a maximum at $j^* \leq \lceil n\pi \rceil$. If $\pi < \frac{1}{n}$, it is decreasing for $j \geq 1$ and, if $\pi > \frac{n(n-1)}{1+n(n-1)}$, it is increasing throughout.*

see Appendix for proof.

The properties of this function will be useful in establishing many of our results.

3 Outreach: When do group loans improve access to credit?

Case 1: No social sanctions

We begin by characterizing the largest loans available under individual and group contracts respectively in the absence of social sanctions.

Recall from (3) that $\rho > K$, so with $\gamma = 0$, the constraint in (4) becomes

$$\frac{n}{j}\frac{r}{B(j)}L \leq K \tag{5}$$

The largest feasible group loan L_g is therefore given by

$$L_g = \frac{j^*B(j^*)}{nr}K \tag{6}$$

where j^* is defined above by Lemma 1. Our first result is that in the absence of social sanctions group loans are always smaller than those offered to individuals.

Proposition 1. *In the absence of social sanctions, the maximum loan size available to a group member, L_g is strictly smaller than L_i, the largest loan available under an individual contract.*

Proof. The largest loan available under an individual contract is given by $L_i = \frac{K\pi}{r}$ as seen in equation (1). This is not feasible as a group loan if, for all $0 < j \leq n$,

$$\frac{n}{j}L_i\frac{r}{B(n,j,\pi)} > K.$$

Using (1), K can be expressed in terms of L_i and we can rewrite the above condition as

$$n\pi > jB(n,j,\pi) \tag{7}$$

We show that this is always true. The LHS is the expectation of a Binomial distribution with parameters n and π and can be expressed as:

$$n\pi = \sum_{k=0}^{n} k\pi_k > \sum_{k=j}^{n} k\pi_k \geq j\sum_{k=j}^{n}\pi_k = jB(n,j,\pi). \tag{8}$$

$\square$

Proposition 1 implies that, in the absence of social sanctions, joint liability cannot allow a person to undertake the project as part of a group if she is denied the loan as an individual. Joint liability is therefore associated with lower outreach than individual contracts. This result is driven by the fact that successful projects under group loans do not always repay the amount K that they would pay under the largest individual loan. On the one hand, if less than j projects succeed, successful projects do not repay anything to the bank. This is captured by the first inequality in the expression (8) above. On the other hand, if more than j projects succeed, each success repays less than K to the bank, since the total amount repaid by the group is equal to jK. This is captured by the second inequality in the expression (8) above.

3.1 Case 2: Social sanctions in groups

Social sanctions improve repayment incentives by relaxing (4) and increases in these sanctions can raise L_g until the point at which $\rho = K + \gamma$. At this point, the entire surplus from successful projects can be extracted and the marginal effect of higher sanctions on loan size is zero. With large enough social sanctions, repayment incentives do not restrict the size of group loans. The question is whether, when the group experiences the smallest number of successes required for repayment, project returns are large enough for succesful members to be able to reimburse the loan. We have:

Proposition 2. *If the level of social sanctions γ is greater than $\frac{(1-2\pi)K+r}{\pi}$, the largest individual loan can always be implemented as a group loan if $n\pi$ is an integer and $\pi \leq \frac{1}{2}$. Moreover, if $\pi < 1 - \frac{r}{K}$, households can always obtain group loans to invest in the project, independent of their initial level of wealth.*

Proof. When $\gamma \geq \frac{(1-2\pi)K+r}{\pi}$, the group can extract at least $\bar{\rho}$ from each successful member by threatening to sanction them: $\bar{\rho} \leq K + \gamma$. To prove the first part of Proposition 2, we need to show that, even if the gross returns to the project are as low as $\bar{\rho}$, one can always find a group contract lending L_i to each member such that, with j successes, the returns to the project allows each successful member to repay the group loan.

When $n\pi$ is an integer, the median, m, is always equal to $n\pi$ (Kaas and Buhrman, 1980). Let us consider the contract for which $j = n\pi = m$. At $L_g = L_i = \frac{\pi K}{r}$, the maximum repayment per successful borrower is given by:

$$\frac{n}{j}\frac{r}{B(j)}L_i = \frac{n}{m}\frac{\pi}{B(m)}K = \frac{K}{B(m)},$$

and earnings are at least equal to:

$$\bar{\rho} = \frac{1-\pi}{\pi}K + \frac{r}{\pi} \geq \frac{1-\pi}{\pi}K + \frac{r}{\pi}L_i = \frac{K}{\pi}. \tag{9}$$

By definition of the median, $B(m) \geq \frac{1}{2}$. When $\pi \leq \frac{1}{2}$, $B(m) \geq \pi$, and, at $j = m$, $\frac{n}{j}\frac{r}{B(j)}L_i \geq \bar{\rho}$. Under the group contract, earnings on successful projects are always large enough to allow repayment of the group loan.

The second part of the proposition examines the condition under which $L_g = 1$ can be implemented as a group loan, when social sanctions are large enough. We again consider a group contract with $j = n\pi = m$, and require earnings to be large enough to allow successful borrowers to repay the loan:

$$\frac{n}{j}\frac{r}{B(j)}L_g = \frac{n}{n\pi}\frac{r}{B(m)} < \bar{\rho} = \frac{1-\pi}{\pi}K + \frac{r}{\pi}$$

$$\Longleftrightarrow \frac{1-B(m)}{B(m)}r < (1-\pi)K.$$

This last inequality holds if $\pi < 1 - \frac{r}{K}$ since $B(m) \geq \frac{1}{2}$. Note finally that, since by assumption $L_i = \frac{\pi K}{r} < 1$, this condition can only be satisfied if $\pi < \frac{1}{2}$. □

The first part of the Proposition states that, with large enough social sanctions, the largest individual loan can always be implemented as a group loan if the probability of success of the project is small enough. The intuition behind this result is that risky projects must have high returns when they are successful so that the participation constraint is satisfied. Social sanctions, if large enough, allow these returns to be extracted from successful members to repay the group loan. The second part of the Proposition indicates that, when the risk of project failure is large enough, large enough social sanctions allow projects to be fully financed by group loans. The intuition behind this result is very similar: since project sucess is unlikely, succesful projects yield large enough returns to repay for the group loan.[7]

Interestingly, for groups of size 2, the largest individual loan can always be implemented with large enough social sanctions. Indeed, if a group contract requires 2 successes for repayment, the probability of group success is equal to π^2. The amount to be repaid by each successful member for a loan of size L_i is given by $\frac{n}{j}\frac{r}{B(j)}L_i = \frac{2}{2}\frac{r}{\pi^2}L_i = \frac{K}{\pi}$. This amount is always strictly smaller than the minimum return in case of success $\bar{\rho}$ (see equation (9)).

However, for other parameter values, the effect of social sanctions on the maximum size of a group loan is much more limited and the maximum loan under a group contract is necessarily below that of an individual contract, irrespective of the level of social sanctions. Under these conditions, social sanctions cannot improve access to credit beyond what would occur under individual contracts. The proposition below characterizes some of the cases under which social sanctions cannot improve outreach.

[7]The proposition requires $n\pi$ to be an integer. This is a technical condition for the proof, and numerical simulations show that the proposition holds for all $\pi \leq \frac{1}{2}$.

Proposition 3. *If $\pi > \frac{n(n-1)}{1+n(n-1)}$, there always exist parameter values for which the maximum group loan is less than the largest individual loan even with arbitrarily large social sanctions.*

Proof. Recall first that, when $\pi > \frac{n(n-1)}{1+n(n-1)}$, the function $jB(j)$ is maximized at $j^* = n$, with $B(j^*) = \pi^n$. If the group loan is equal to the largest individual loan L_i, the largest amount a successful member in the group has to reimburse is given by:

$$\frac{n}{j}\frac{r}{B(j)}L_i = \frac{r}{\pi^n}L_i,$$

while the minimal return to the project $\bar{\rho}$ can be written as:

$$\bar{\rho} = \frac{1-\pi}{\pi}K + \frac{r}{\pi} = \frac{1-\pi}{\pi}\frac{rL_i}{\pi} + \frac{r}{\pi} < \frac{1-\pi}{\pi}\frac{r}{\pi} + \frac{r}{\pi} = \frac{r}{\pi^2}.$$

Using these two expressions, we have:

$$\frac{n}{j}\frac{r}{B(j)}L_i > \bar{\rho} \text{ if } L_i \geq \pi^{n-2}.$$

Under this condition, group loans proposing the largest individual loan are not feasible, as successful members are never able to repay the loan out of the returns of their projects. □

When π is large, we can easily find situations under which the largest individual loan is not feasible as a group loan. The proposition above illustrates one of these situations, but is clearly not a complete description of these cases.[8] The intuition behind the result is that, when the expected return to the project is low and the risk of project failure is also low, the returns in case of success are close to the expected returns. As a result, even with very large social sanctions, groups cannot extract much more from successful projects than the banks. By contrast, the risk of group failure is still sizeable, so that the amounts to be repaid by successful members exceeds what has to be paid under individual contracts.

[8] For instance, it can easily be shown that L_i is not feasible as a group loan if parameter values are such that $\rho = \bar{\rho}, L_i \geq 0.914, \pi > 0.779, n = 5$ or $\rho = \bar{\rho}, L_i \geq 0.858, \pi > 0.936, n = 10$...

3.2 Outreach: comparing bank and social sanctions

As discussed above, outreach under group lending depends on the strength of bank and social sanctions. Even though, in the absence of more elaborate micro-foundations, bank sanctions and social sanctions are not strictly comparable, it is interesting to investigate which type of sanctions is the most effective in inproving outreach. We consider a hypothetical situation under which bank and social sanctions can be increased by the same amount, and focus on their relative impact in terms of access to loans. We have:

Proposition 4. *An increase in social sanctions is always less effective in increasing outreach than a similar increase in bank sanctions. It is strictly less effective if* $L_i \geq L_g$.

Proof. First note that, since $\rho \geq \bar{\rho}$, the largest individual loan is given by the incentive compatibility condition (1): $L_i = \frac{\pi K}{r}$. We thus have:

$$\frac{\partial L_i}{\partial K} = \frac{\pi}{r}.$$

As before, the largest group loan is defined by $\frac{n}{j^*} \frac{r}{B(j^*)} L_g = \min(\rho, K + \gamma)$. Two situations can therefore occur. In a first situation, the largest group loan is given by the resource constraint: $\frac{n}{j^*} \frac{r}{B(j^*)} L_g = \rho$. Changes in sanctions from the bank or from the group have no impact in this case. In a second, more interesting situation, the repayment incentive constraint is binding and $K + \gamma > \rho$. We then have:

$$\frac{\partial L_g}{\partial K} = \frac{\partial L_g}{\partial \gamma} = \frac{j^* B(j^*)}{nr}.$$

Proposition 1 shows that $j^* B(j^*) < n\pi$, which implies:

$$\frac{\partial L_g}{\partial K} = \frac{\partial L_g}{\partial \gamma} < \frac{\partial L_i}{\partial K}.$$

When $L_i \geq L_g$, bank sanctions increase outreach by increasing L_i at a more rapid rate then L_g. If $L_i < L_g$, a marginal increase in bank or social sanctions have exactly the same impact on outreach since K and γ enter symmetrically in the repayment incentive constraint. □

When the largest loan available is through an individual contract, increasing bank sanctions is much more effective in increasing outreach than social sanctions. This is because, through the repayment incentive constraint, the multiplier effect associated with sanctions is always larger under individual loans than under group loans. This property was already the fundamental mechanisms underlying Proposition 1, through which bank sanctions have a larger impact (in terms of size) on individual loans than on group loans. The above Proposition is a consequence of this.

4 The benefits from joint liability:

4.1 Comparing group loans and individual loans

We now investigate the benefits from group contracts as compared to the individual contracts. For any wealth level w which requires bank financing, the minimum loan size needed for investment is given by $(1-w)$. Call this $L(w)$. For a group lending contract to be feasible for loan size $L(w)$, we require that $L(w) \leq L_g$. Given n and π, we let $\underline{j}$ be the smallest number of successes such that the following inequality is satisfied:

$$\frac{n}{j} L(w) \frac{r}{B(j)} \leq K + \gamma. \tag{10}$$

By definition, $\underline{j}$ corresponds to the lowest probability of group failure, and therefore to the lowest interest rate charged on that loan, $\frac{r}{B(\underline{j})}$. By Lemma 1, $\underline{j}$ is also increasing with the amount borrowed, L.

The utility to an individual from a group loan of size L depends both on whether the individual's project has succeeded or not and whether the group has enough successes to repay the bank. The expression for the expected utility per member can therefore be written as the sum of two terms, one for the case of group default and the other for repayment.

If $j < \underline{j}$, the bank will always sanction group members and the individual member will keep his return ρ if the project happens to be successful, which happens with probability $\frac{j}{n}$. When $j > \underline{j}$, group repayment occurs, there are no bank sanctions, and each successful member pays $\rho - \frac{n}{j}\frac{r}{B(\underline{j})}L$ to the bank. The expected utility from a feasible group contract is therefore

$$U_g = \sum_{k=0}^{\underline{j}-1} \pi_k \left(\frac{k}{n}\rho - K - rw\right) + \sum_{k=\underline{j}}^{n} \pi_k \left(\frac{k}{n}\left(\rho - \frac{n}{k}\frac{r}{B(\underline{j})}\right) - rw\right)$$

Using the definition of $B(\underline{j}) = \sum_{k=\underline{j}}^{n} \pi_k$, we simplify the above expression to obtain:

$$U_g(\underline{j}) = \pi\rho - r - K \sum_{k=0}^{\underline{j}-1} \pi_k = \pi\rho - r - \left(1 - B(\underline{j})\right) K \tag{11}$$

This expression is a function of the minimum number of successes $\underline{j}$ required to repay the loan. Since $\underline{j}$ is increasing in loan-size and $U_g(\underline{j})$ is decreasing in $\underline{j}$, groups with lower initial wealth have lower benefits from group loans. In other words, the individuals who least require group loans are those who benefit most from them. This discussion is summarized in the following proposition:

Proposition 5. *The benefits from group contracts are increasing in borrower wealth.*

We can now compare individual and joint liability contracts for the same loan size. Using (2) and (11), we obtain:

$$U_g - U_i = [B(\underline{j}) - \pi]K \tag{12}$$

The expected gain from group relative to individual contracts is simply equal to the difference in the probability of being sanctioned by the bank. It is true that successful members in groups pay more than they would under an individual contract when there are fewer than $n\pi$ successes in the group, but these additional payments are offset by the occasions on which other group members pay off their loans.

It is worth emphasizing here that the expression in (12) needs not be positive. It may well be that a group loan is feasible, in that groups have the correct incentives to repay, yet the expected utility of each their members is higher under an individual contract. This happens when, at the loan size required by the group, the number of success required for repayment $\underline{j}$ are high enough to result in $B(\underline{j}) < \pi$. However, there always exists a level of wealth above which group loans are strictly preferred to individual loans. Indeed, for high enough wealth levels, only one successful project in the group is needed to repay the group loan. To find the range of wealth levels for which this is true, notice that the probability of repayment by the group if only one success is needed is given $B(1) = (1-\pi)^n$. Using (10), the loan amount for which the group requires only one success for repayment is equal to $\frac{K(1-\pi)^n}{nr}$. For all individuals with a level of initial wealth at least equal to $w_h = 1 - \frac{K(1-\pi)^n}{nr}$, the expected gain from a group loan over an individual loan is given by:

$$U_g - U_i = [1 - (1-\pi)^n - \pi]K = [(1-\pi) - (1-\pi)^n]K > 0$$

We can now describe more fully the equilibrium distribution of group and individual contracts for different levels of wealth. First consider a situation where j^* (which maximizes $jB(j)$) is such that $B(j^*) \geq \pi$ and there is no social sanctions. Following Proposition 1 above, we know that L_g is then smaller than L_i. In this case, all individuals with a level of wealth above $(1 - L_g)$ participate in groups and borrow $(1 - w)$. Since richer individuals need to borrow less, they belong to groups requiring fewer successes and derive more utility from their investment. Individuals with a level of wealth w such that $(1 - L_g) > w \geq (1 - L_i)$ take an individual loan. Individuals with a lower level of wealth do not borrow and do not invest.

Allowing for positive social sanctions, more individuals are able to take up a group loan, though individual loans are still chosen by the poorest investors. If social sanctions become very large and the conditions defined in Proposition 2 are satisfied, all eligible individuals choose a group loan (since $B(j^*) \geq \pi$) and individual loans are not offered in equilibrium. The case where $B(j^*) < \pi$ can be discussed in a similar way. The interesting difference is that the individuals who are eligible for the largest group loans will opt for an individual loan as they derive a higher utility from it. With large enough social sanctions and under the conditions defined by Proposition 2, group loans will be chosen either by the wealthier individuals (for which $B(j) \geq \pi$), or by the poorest ones who are eligible for a group loan but not for an individual one.

4.2 Welfare implications of variations in group size

We cannot provide a complete characterization of the impact of group size on outreach or the welfare of group members. The problem arises because, for groups of relatively small size, while the probability of having at least j successes is increasing in the number of members in the group, the probability of having at least a proportion j/n successes does not necessarily. However, we can show that the impact of group size on the welfare of group members can be positive or negative, depending on the value of the parameters, and in particular on the minimum number of successes required to repay the group loan.

Theorem 4.1.

Proposition 6. *For sufficiently small loan sizes, the gains from group lending are decreasing as group size falls.*

Proof. We know from Proposition (...) that the gain from group loans depends critically on the repayment probability $B(j)$:

$$U_g(j) = \pi\rho - r - (1 - B(j))(K + \gamma)$$

We focus on groups with w close to 1, and more specifically, we require the following conditions:

$$n(1-w)\frac{r}{B(n,1,\pi)} \leq K + \gamma$$

and

$$(n-1)(1-w)\frac{r}{B(n-1,1,\pi)} \leq K + \gamma.$$

These two conditions state that group members are wealthy enough to take a loan that requires only one success to be repaid in a group of size n and of size $n-1$. Since $B(n,1,\pi)$ is increasing in n, members' welfare falls if the size of the group decreases. □

Wealthy groups need to make only small loans, for which only one success is required to allow repayment. In this case, reducing group size decreases the chances of repayment by the group, and thus reduces members' welfare. Conversely, increasing group size increases welfare as long as only one success is required for group repayment, that is as long as the following condition is satisfied:

$$n(1-w)\frac{r}{B(n,1,\pi)} \leq K.$$

Once n becomes too large, the minimum number of success required switches to more than one, and at that point, welfare falls, though it may rise again with further increases in n.

We now show that, if π is large enough so that, for the largest group loan, all members are required to succeed, welfare may fall with increases in group size. For the problem to be well-defined, we require that the group loan is largest when $j^* = n$. As discussed in Section (...), this holds if $\pi \geq \frac{n(n-1)}{1+n(n-1)}$. Moreover, we know that, when a group loan requires n successes, the individual loan of corresponding size will be preferred, as the probability of failure, and therefore of bank sanctions, is lower for individual loans. Therefore, a group loan requiring n successes will only be chosen if no individual loans of that size are available, and if project returns are large enough. For those group loans, we have:

Proposition 7. *If* $\pi \geq \frac{n(n-1)}{1+n(n-1)}$, $\rho \geq \frac{1-\pi^n}{\pi}K + \frac{r}{\pi}$ *and* $\frac{K+\gamma}{K} > \pi^{1-n}$, *a fall in group size increases the welfare of the members requiring the largest loan and increases outreach.*

Proof. First note that, since $\pi \geq \frac{n(n-1)}{1+n(n-1)}$, $j^* = n$ for a group of size n. In a group of size $(n-1)$, $j^* = n-1$, since, if $\pi \geq \frac{n(n-1)}{1+n(n-1)}$, $\pi \geq \frac{(n-1)(n-2)}{1+(n-1)(n-2)}$. Second, when $\frac{K+\gamma}{K} > \pi^{1-n}$, $\frac{L_g}{L_i} = \frac{B(n)}{\pi}\frac{K+\gamma}{K} > 1$ so that $L_g > L_i$. Third, L_g is individually rational in a group of size n if $\rho \geq \frac{1-\pi^n}{\pi}K + \frac{r}{\pi}$, so that individuals prefer to take such a loan than none.

Consider now, under the same conditions, the largest loan available for a group of size $(n-1)$, $L_g(n-1)$. Since, under a binomial distribution, $B(n,n,\pi) < B(n-1,n-1,\pi)$, we have:

$$L_g(n-1) = \frac{(n-1)B(n-1)}{(n-1)r}(K+\gamma) > \frac{nB(n)}{nr}(K+\gamma) = L_g(n).$$

This condition indicates that outreach is larger under groups of size $(n-1)$ compared to groups of size n. Moreover, it implies that members in a group of size n can still borrow the same amount, $L_g(n)$, in a group of size $(n-1)$. But, since $B(n,n,\pi) < B(n-1,n-1,\pi)$, they also strictly prefer the smaller group. □

5 Discussion and conclusions

In this paper, we have investigated the consequences of joint liability for access to loans by the poor. As we have shown, in the absence of social sanctions, joint liability does not allow group loans to be offered to those excluded from the more traditional sources of credit. Social sanctions may help in this respect, by better enforcing repayment by group members, even though, in a number of cases, social sanctions do not improve access. However, we also show an increase in bank sanctions is more effective than a similar increase in social sanctions in improving access to credit by the poor. In this sense, an improvement in the legal environment of traditional credit sources may result in more benefits to the poor than a policy reinforcing group lending and social sanctions.

From a distributive point of view, we also showed that the distribution of the benefits from group lending are skewed in favour of the wealthiest groups who need to borrow small amounts. In this sense, the availability of group loans has anti-redistributive properties by favoring those who need them less. Finally, while we could not provide a complete characterization of the optimal size of a group, we have shown that increasing group size may increase welfare for groups that need to borrow very little, while it may reduce welfare in groups that need to borrow a lot.

5.1 Appendix

Proof of Lemma 1

Note first that $\lceil n\pi \rceil \geq 1$ for $\pi > 0$ and the function $jB(j) = n\pi^n$ for $j = n$. From the properties of the binomial distribution, we also know that both the mode and the median of a binomial distribution are at most $\lceil n\pi \rceil$ and at least $\lfloor n\pi \rfloor$ (Kass and Buhrman, 1990).

We now show that the function $jB(j)$ is decreasing to the right of $\lceil n\pi \rceil$. Consider $n > j \geq \lceil n\pi \rceil$. The function $jB(j)$ is strictly decreasing at j if $jB(j) > (j+1)B(j+1) \iff jB(j+1) + j\pi_j > jB(j+1) + B(j+1) \iff j\pi_j > B(j+1)$.

By definition of the binomial probabilities, we know that

$$\pi_j = \pi^j (1-\pi)^{n-j} \frac{n!}{j!(n-j)!}$$

and

$$\pi_{j+1} = \pi^{j+1} (1-\pi)^{n-j-1} \frac{n!}{(j+1)!(n-j-1)!}.$$

Combining these two expressions, we have

$$\begin{aligned} j\pi_j &= \left(\frac{j}{n-j} \frac{1-\pi}{\pi} \right) (j+1)\, \pi_{j+1} \\ &= \left(\frac{j}{n-j} \frac{1-\pi}{\pi} \right) \pi_{j+1} + j \left(\frac{j}{n-j} \frac{1-\pi}{\pi} \right) \pi_{j+1} \end{aligned}$$

But $\frac{j}{n-j} \frac{1-\pi}{\pi} \geq 1$ if $j \geq n\pi$, which implies that:

$$j\pi_j \geq \pi_{j+1} + j\pi_{j+1}. \tag{13}$$

Following the same steps, we can express $j\pi_{j+1}$ as a function of π_{j+2}:

$$j\pi_{j+1} = \left(\frac{j}{n-j-1} \frac{1-\pi}{\pi} \right) (j+2)\, \pi_{j+2}.$$

Since $\frac{j}{n-j-1}\frac{1-\pi}{\pi} > 1$ for $j \geq n\pi$, we have:

$$j\pi_{j+1} > \pi_{j+2} + j\pi_{j+2}. \tag{14}$$

Combining (13) and (14), we therefore have:

$$j\pi_j > \pi_{j+1} + \pi_{j+2} + j\pi_{j+2}.$$

For all $k \leq n-1$, we can continue expressing $j\pi_k$ as a function of π_{k+1}, and we obtain:

$$j\pi_j > \sum_{k=j+1}^{n} \pi_k = B(j+1).$$

As a result, $jB(j)$ is monotonically (strictly) decreasing to the right of $\lceil n\pi \rceil$.

We then show that the function $jB(j)$ has an inverted-U shape to the left of $\lceil n\pi \rceil$. Note first that the function $jB(j) = 0$ for $j = 0$, and $jB(j) > 0$ for $j > 0$, so that the function $jB(j)$ is increasing at $j = 0$. Consider $j : 1 \leq j \leq \lfloor n\pi \rfloor$. Let us first consider the case where $jB(j) \leq (j+1)B(j+1) \iff j\pi_j \leq B(j+1)$. Since the mode M is greater or equal to $\lfloor n\pi \rfloor$, and π_k is strictly increasing for $0 \leq k \leq M$, we have $\forall z < j, \pi_z < \pi_j$ which implies $z\pi_z < j\pi_j$. However, $B(z+1) > B(j+1)$. We therefore have for $z < j$:

$$z\pi_z < j\pi_j \leq B(j+1) < B(z+1)$$

so that $zB(z) < (z+1)B(z+1)$for all $z < j$. Let us now consider the case where, for a given value of $j \leq \lfloor n\pi \rfloor$, $jB(j) \geq (j+1)B(j+1) \iff j\pi_j \geq B(j+1)$. $\forall z : j < z \leq M, z\pi_z > j\pi_j \geq B(j+1) > B(z+1)$, which implies that $zB(z) > (z+1)B(z+1)$ for all $z > j$.

We can now turn to the second part of the proposition. The function $jB(j)$ is maximised at $j = 1$ if $\lceil n\pi \rceil \leq 1$, which holds if $\pi \leq \frac{1}{n}$. As a result, if $\pi < \frac{1}{n}$, $jB(j)$ is decreasing for $j \geq 1$. The function $jB(j)$ is maximised at $j = n$ if $M \geq n-1$ and if $nB(n) > (n-1)B(n-1)$. We first examine the second condition:

$$\begin{aligned} nB(n) &> (n-1)B(n-1) \\ \iff n\pi^n &> (n-1)\left(\pi^n + n\pi^{n-1}(1-\pi)\right) \\ \iff \frac{n}{n-1} &> 1 + n\left(\frac{1-\pi}{\pi}\right) \end{aligned}$$

$$\iff \pi > \frac{n(n-1)}{1+n(n-1)}. \tag{15}$$

However, at the values of π satisfying the above condition, $n\pi > n-1$, so that $M \geq n-1$.

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